Deconstructing Delay Dynamics

An air traffic network example

Karthik Gopalakrishnan and Hamsa Balakrishnan
Department of Aeronautics and Astronautics
Massachusetts Institute of Technology
Cambridge, MA, USA
{karthikg,hamsa}@mit.edu

Richard Jordan
Group 43, Air Traffic Control Systems
MIT Lincoln Laboratory
Lexington, MA, USA
richard.jordan@ll.mit.edu

***Abstract*—This paper develops and analyzes a simplified model of the dynamics of delay propagation in air traffic networks. The proposed model considers the redistribution of delays by accounting for aircraft flows between airports, the persistence of delays at an airport, the mitigation of delays due to slacks or buffers in flight schedules, and inputs such as sudden impulsive disruptions or sustained impacts due to longer duration traffic management initiatives. Using inter-airport traffic flows from operational data, different properties of the model are studied, including the resilience of different airports (as measured by the length of time before delays mitigate after a disruption), the amount of delay induced by disruptions at a particular airport, and the number of airports that are impacted when a given airport experiences disruptions. These properties are evaluated for different levels of delay sustainment, and for different values of available slack in schedules.**

Keywords- delay propagation; network modeling; system performance; resilience

I. Introduction

The air transportation system has evolved into a large-scale, interconnected network with many interacting elements. As a result of the large number of shared airport and airspace resources, disruptions in one part of the system can propagate to many others. A significant portion of these propagations occur at airports (that is, the nodes of the air transportation network), where incoming aircraft continue on the subsequent legs of their planned itineraries, crew members may connect to other flights, and passengers also connect to other flights. Aircraft connectivity is known to be a key driver of flight delays; nearly one-third of all delayed domestic departures (and 40% of departure delays in minutes) in the United States are due to the late arrival of the aircraft on its previous leg. Air carrier delay (a category which includes crew connections) is the associated cause for another 28% of delayed departures (and 32% of delays in minutes) [1]. The above statistics suggest that flows of aircraft and crew through airports are the dominant mechanism by which delays propagate through the system. This paper therefore proposes a model that relates traffic flow in the air traffic network to the flow of delays, and uses the model to analyze the dynamics of delays on the network.

*This work was partially supported by NASA under Air Force Contract No. FA8721-05-C-0002, and by NSF under CPS:Frontiers:FORCES, grant number 1239054. Opinions, interpretations, conclusions, and recommendations are those of the authors and are not necessarily endorsed by the United States Government.

Network models have been previously proposed for a vast range of problems, from disease epidemics [2] and rumor propagation [3], to engineered systems such as power grids [4, 5], the Internet [6], roads [7], public transport [8], railroads [9] and even air transportation [10]. They have also been used to evaluate connectivity between airports in terms of operations [11, 12, 13]. Prior research on spreading processes in networked systems has focused primarily on epidemiological models [14]. These models typically assume that a node is in one of a small set of discrete-states; by contrast, air traffic delays are better modeled as continuous variables. Epidemiological models are generally based on undirected, unweighted networks; however, air traffic delay networks are weighted and directed in nature [15].

Numerous prior studies on modeling air traffic delay propagation [16, 17, 18, 10, 19, 20, 21, 22, 23, 24, 25, 26] have revealed the underlying complexities of the process, and the inherent challenges in predicting system behavior [27, 28, 29, 30, 31]. These complexities include the characteristic that while traffic flows through airports result in a spread of delays, the build-up of queues can result in a persistence of delays even after aircraft depart from the airport (or a weather disturbance subsides), and the buffers or slacks contained within flight schedules can help mitigate delays (that is, remove delays from the system). Another challenge is that the interactions between different pairs of airports occur at different time-scales due to differences in flight times between them. For example, it may only take an hour for delays to propagate from Boston to an airport in New York City, while it may take several hours for delays at Boston to propagate to San Francisco. This paper proposes a model that allows for these phenomena, while accounting for the fact that delay propagation is primarily driven by traffic flows between airports.

While the proposed model can be adapted to any given structure of traffic flows between airports (or even delay flows [15]), an illustration using traffic demand data from the Bureau of Transportation Statistics [1] for 2011 – in particular, a network whose edges are weighted by the average number of daily flights between two airports – is presented as a proof-of-concept. Different system performance characteristics, including the resilience of different airports (as measured by the length of time before delays mitigate after a disruption), the delays induced by disruptions at different airports, and the number of airports that are impacted when a given airport experiences disruptions, can all be evaluated and compared using the proposed modeling approach.

II. Model of delay dynamics

We model the air traffic network as a weighted directed network with N *vertices* (nodes or airports). Each *edge* is represented as an ordered pair, (v_1, v_2), denoting a link from v_1 to v_2. Edge (i, j) is assumed to have a nonnegative weight associated with it, representing the flow of traffic from one node to another. The adjacency matrix is given by $\Theta = [\theta_{ij}]$, where each element θ_{ij} denotes the number of flights from airport i to airport j. Fig. 1 shows the network of average daily traffic in 2011.

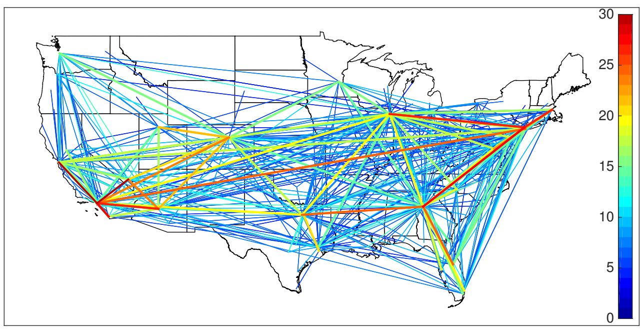

Fig. 1: Average daily traffic in 2011 [1]. The color denotes the average number of flights on each link

The state of airport i at time-step t, denoted $x_i(t)$, is defined as the average delay per flight at airport i at time t. The state vector of the system at time-step t is therefore given by

$$\vec{x}(t) = \begin{bmatrix} x_1(t) & x_2(t) & \cdots x_N(t) \end{bmatrix}.$$

A. Characteristics of air traffic delay dynamics

The proposed model of delay dynamics must reflect the following characteristics exhibited by air traffic delays:

- **Redistribution of delays**: The basic premise of the proposed model is that delays at an airport tend to be redistributed amongst the traffic traversing through it. For example, consider a particular airport where there are 10 flights arriving with a delay of 25 min/flight and 5 flights arriving with a delay of 10 min/flight. In the absence of any other effects (such as slack in the schedules), if the outbound delay equals the inbound delay, then the average delay of a flight leaving the airport will be $\frac{10\times25+5\times10}{10+5} = 20$ min/flight. If there are two outbound links (going to airports k and l, respectively, with 5 flights on each route), then the network will tend to redistribute 10 min/flight to each of the airports k and l.
- **Persistence of delays**: Another attribute of air traffic delays is their tendency to persist at an airport, even after the disruption has ended. One reason for this phenomenon is the build-up of queues or workload that then takes time to subside. The proposed model reflects this behavior by assuming that a fraction $\alpha \in [0, 1]$ of the delay level at an airport at any time persists through the next time-step. In the current model, we set $\alpha_i = \alpha\ \forall i$. As α decreases from 1 to 0, the inertia at airports (persistence of delays) increases.
- **Slack in the system**: Flight schedules are known to contain some amounts of slack or buffering that can mitigate delay propagation to a certain extent [17]. This slack take the form of longer-than-necessary block times (that is, schedule padding) or long turnaround times between two consecutive legs of an aircraft itinerary. In either form, the available slack in the schedule prevents the propagation of delays that are within the amount that can be handled by the buffer. This attribute is modeled by a slack term of β min/flight on each link of the network. This term serves to decrease the propagation of delays through a link.
- **Multiple time scales**: Because distances (and therefore flight times) over different links vary, airport delays interact over multiple time scales. In other words, while it may take 1 hour for delays to propagate from Boston (BOS) to New York's LaGuardia (LGA) airport, while it may take several hours for them to propagate from BOS to Dallas/Fort Worth (DFW) airport. The network model is augmented with pseudo-nodes to model these variations in time-scales. In our discrete-time model, each time-step corresponds to 1 hour; it should take multiple time-steps for delays to propagate between two airports that are more than an hour's flight time apart. The augmented network is created by inserting pseudo-nodes between airports that are more than 1-hour apart. If the transit time along the edge from node i to j is h hours, then we introduce a chain of $h-1$ pseudo-nodes between them, each 1-hour from the next. The traffic on each of these edges will be θ_{ij}. If P pseudo-nodes are added, the augmented network contains $V = N + P$ nodes. The adjacency matrix of the augmented matrix is denoted $A = [a_{ij}] \geq 0$, $A \in \mathbb{R}^{V\times V}$. Fig. 2 shows the original and augmented networks for the case when one of the edges has a transit time of 2 hours and another has a transit time of 3 hours.

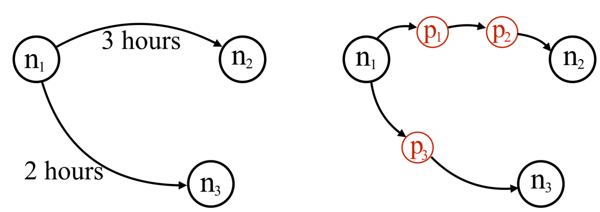

Fig. 2: (Left) Original network. The flight time from node n_1 to n_2 is 3 hours, and from node n_1 to n_3 is 2 hours. (Right) The augmented network has three pseudo-nodes p_1, p_2 and p_3

Suppose the traffic matrix is given by

$$\Theta = \begin{bmatrix} 0 & \theta_{12} & \theta_{13} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

The augmented network has 6 vertices, $V = \{n_1, n_2, n_3, p_1, p_2, p_3\}$ and its adjacency matrix is given by

$$A = \begin{bmatrix} 0 & 0 & 0 & \theta_{12} & 0 & \theta_{13} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \theta_{12} & 0 \\ 0 & \theta_{12} & 0 & 0 & 0 & 0 \\ 0 & 0 & \theta_{13} & 0 & 0 & 0 \end{bmatrix}.$$

- **Exogenous input**: The injection of delay into the system at an airport i is assumed to take the form of an exogenous input, $u_i(t)$. This delay could be caused by bad weather or other disruptions, or because of traffic management initiatives (such as ground delays or ground stops) that are issued in response to such disturbances. We assume that this input is non negative.

B. Governing equations

The model features described in Sec. II-A can be combined to determine the equations that govern the evolution of the state vector, $\vec{x}(t)$. First, we note the presence of two factors: the persistence of delays at an airport, and the redistribution of delays due to network effects. We assume that the delay at any time-step is a convex combination of these two factors at the previous time-step. As α increases and delays persist from one time-step to the next, the influence of network effects decreases. Second, we note that the network effects term represents the average delay level of the airport and depends on the incoming delay, which in turn depends on the incoming traffic flows. However, some of the incoming delay is mitigated by of the slack on each link leading into that node. The pseudo-nodes are assumed to just transfer the delay along incident edges. The resultant equations for the evolution of delays are described by (1)-(2):

$$x_i(t+1) = \alpha x_i(t) + (1-\alpha)\frac{\sum_j a_{ji}\ \left(x_j(t)-\beta\right)^+}{\sum_j a_{ji}} + u_i(t), \forall i \in N; \quad (1)$$

$$x_i(t+1) = \sum_j \mathbb{I}_{a_{ij}>0}\, x_j(t), \forall i \in P, \quad (2)$$

where $(x_j(t)-\beta)^+ = \max\{x_j(t)-\beta, 0\}$, and $\mathbb{I}_{a_{ij}>0}$ is an indicator variable which is 1 when $a_{ij} > 0$ and 0 otherwise.

The term $\frac{\sum_{j\in V} a_{ji}\ \max\{x_j(t)-\beta,0\}}{\sum_{j\in V} a_{ji}}$ is the traffic-weighted incoming delay. The max operator ensures that the delays do not become negative; it however results in nonlinear system dynamics. The exogenous input is given by $u_i(t)$.

Although not considered in this work, an extension of the model would involve airborne delays as well. These delays could be injected via an edge specific input $\tilde{u}_{ij}(t)$. The network effect term would then be $\frac{\sum_{j\in V} a_{ji}\ \left(x_j(t)-\beta+\tilde{u}_{ij}(t)\right)^+}{\sum_{j\in V} a_{ji}}$. $\tilde{u}_{ij}(t)$ can be positive as well as negative and can be used to model delay reductions.

III. Illustrative analysis

The proposed network model can be adapted using operational data from a variety of possible sources, such as the Aviation System Performance Metrics (ASPM) database [32] and the Bureau of Transportation Statistics (BTS) database [1]. The primary difference is that the BTS data only includes records of US carriers which accounted for at least 1% of passenger revenues, and therefore does not include airports that are served by smaller carriers, air taxis, etc. We illustrate our approach through an analysis of the model developed using operational air traffic data from BTS for the year 2011 [1]. We only consider links (Origin-Destination pairs) that have at least 5 flights per day. The rationale for not including the smaller links is that there is likely to be sufficient slack on these routes that delays would tend not to propagate through them, that is, their contribution to delays at other airports would remain negligible. The resultant traffic network contains 158 airports and 1,102 links, as shown in Fig (1). The top 10 airports in this network (in terms of traffic between the nodes) are listed in Table I. We note that these counts of daily departures are smaller than the daily operational counts from other databases (such as OPSNET [33]) because they only account for traffic within this network, and not international flights, smaller/international air carriers, and flight legs with infrequent service.

TABLE I
Top 10 airports by traffic in the network model

Airport	Avg. no. of daily departures
Atlanta (ATL)	912
Chicago (ORD)	662
Los Angeles (LAX)	489
Dallas (DFW)	486
Denver (DEN)	468
Phoenix (PHX)	417
San Francisco (SFO)	300
Las Vegas (LAS)	285
Houston (IAH)	279
Charlotte (CLT)	239

We use distance as a proxy for time to determine the augmented network, which has 2,554 nodes (and pseudo-nodes). The average traffic flow matrix for each hour, Θ, is symmetric and is used to construct the adjacency matrix of the augmented network, A.

A. Exogenous inputs and performance metrics

In the subsequent analysis, we use the exogenous input $u_i(t)$ to simulate the injection of delays into a particular node (termed the *inducing airport*) and evaluate the resulting behavior of the system. In particular, we focus on two types of exogenous input functions:

1) *An impulse input*, where we introduce a certain amount of delay at a particular time-step (in this case $t = 0$), and then maintain $u_i(t) = 0\ \forall t > 0$.
2) *A constant set-point*, where we vary the control input in order to keep the delay $x_i(t)$ at a particular airport i at a fixed set-point x_i^*.

These two input functions simulate transient and sustained delay respectively. Although a sustained delay will mean that $u_i(t)$ is a constant, in the steady state it is equivalent to having a constant set point x_i^*. We use the following performance metrics to evaluate the system behavior in response to the exogenous inputs:

1) *Total delay*, namely, the sum of the delay levels seen at all the airports in the network at time-step t, that is, $\sum_{j\in N} x_j(t)$.
2) *Average induced delay*, namely, the average delay level seen across all airports when an exogenous input is introduced at an inducing airport. It is defined as:
$$\bar{\mathrm{ID}}(t) = \frac{\text{total delay}(t)}{|N|},$$
and is the expected delay level that an airport will see under that particular exogenous input and inducing airport.
3) *Largest impacted cluster*, or the largest set of connected airports that have a non-zero delay. If we have an exogenous input only at one airport, then the size of the largest impacted cluster (also known as the giant component) is simply the number of airports with non-zero delay. This is because the dynamics is such that the delay spreads only when there is a connected path. Further, airports with near-zero delay $x_i(t)$ are also counted in this metric. Whenever an airport has a non-zero delay, it means that there is atleast one incoming link with a delay of greater than β min/flt. Consequently, the largest impacted cluster includes airports that have high incoming delays but propagate very little.

B. Impulse input (impulsive disruption)

An impulse input at an inducing airport k is of the form $u_k(0) > 0$ and $u_k(t) = 0\ \forall t > 0$. The exogenous input is assumed to be zero at all other airports. This represents a scenario in which there is sudden brief disruption at an airport. The system response depends on the propensity towards the persistence of delays (that is, α) and the schedule slack (that is, β).

- If $\alpha = 1$, the system is completely inertia-driven. The impulse will be isolated, but persist indefinitely.
- If $\alpha \in (0,1)$ and $\beta = 0$, then there is no slack in the system and delays will disperse through the network, and persist indefinitely. Lemma 1 characterizes this scenario:
  Lemma 1: Consider a system governed by (1)-(2) with $\alpha \in (0,1)$, $\beta = 0$ and an associated symmetric traffic matrix. If we introduce an impulse delay $x_k(0)$ at an airport k, the system will reach a steady-state where the delays at all the airports will be given by
  $$x^{\mathrm{SS}} = \frac{x_k(0)\deg(k)}{\sum_{i\in N}\deg(i)}.$$
- If $\alpha = 0$ and $\beta = 0$, then the delays can keep oscillating and not converge. A simple example is a two-node network where one airport receives an impulse delay. The delay will keep getting transferred between the two airports.
- If $\beta > 0$, then we have slack in the system and delays will get absorbed. In other words,
  Lemma 2: Consider a system governed by (1)-(2) with $\beta > 0$. For any delay impulse, $|x_i(t)| \to 0$ as $t \to \infty$, $\forall i \in N$.

The dynamics for an intermediate range of α and β is simulated for an impulse input delay of 120 min/flight at Chicago O'Hare (ORD) International Airport. In Fig 3, we plot the dynamics of the average induced delay and the size of the largest connected cluster. First, we note that the time for delays to decay to zero increases with α. This increase is nonlinear and grows rapidly as α approaches 1. The parameter α is also related to the response time of the system: When it is low, the system can share the delay with other airports much faster. This distribution enables more flights to use up the slack in their schedule to mitigate the delay. However, when α is low, the peak delay seen is also higher. Thus α determines whether the system will experience a low level of delay for a long period of time or a high level of delay for a short time. The maximum size of the largest impacted cluster is found to be independent of α (and around 70).

With increasing β, the average induced delay and the number of impacted airports both decrease. Comparing it with the effect of α, we observe the following. When we decrease α, the delays reduce and persist for longer, where as when we increase β, the delays scale down proportionally. This highlights the fundamentally different way in which these parameters impact delay dynamics. Fig. 5 shows the time needed for delays to subside, for varying values of α and β, for an impulse input of 120 min/flight at ORD.

C. Constant input (sustained disturbance)

In this section, we study the system behavior under a constant delay input. Since the system is not linear, the superposition principle cannot be used to relate the constant-input response to the impulse-response from Section III-B. This scenario is analogous to one in which a traffic management initiative is used to maintain delays at an airport at a specified level or set-point over a sustained period of time. The initial condition is assumed to be $x_i(0) = 0\ \forall i \in N$. An appropriate "exogenous input" can then be engineered to maintain a set point of x^* for a particular airport k.

When $\beta = 0$, there is no mechanism for delays to be absorbed. The steady state solution will have all airports at a delay of x^*. When $\beta > 0$, then there is some slack in the system it will reduce the exposure of other airports to the delay input. We look at ORD to study the influence of the slack parameter β and the set point x^*. Fig. 7 shows the time

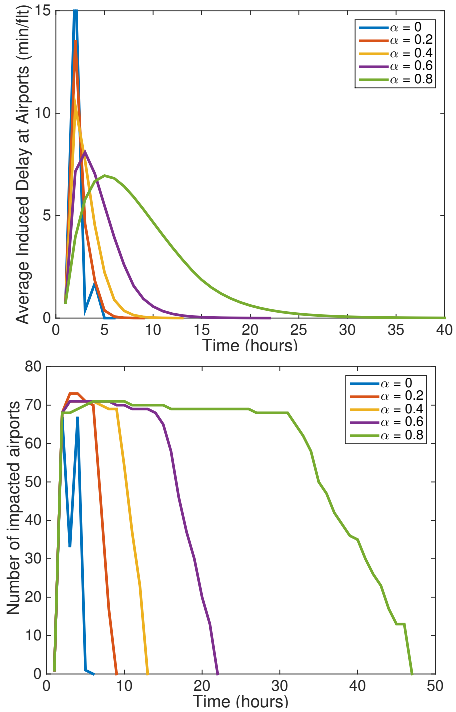

Fig. 3: (Top) Avg. induced delay, and (bottom) number of impacted airports for varying α, for an impulse input of 120 min/flight at ORD and $\beta = 10$ min/flight

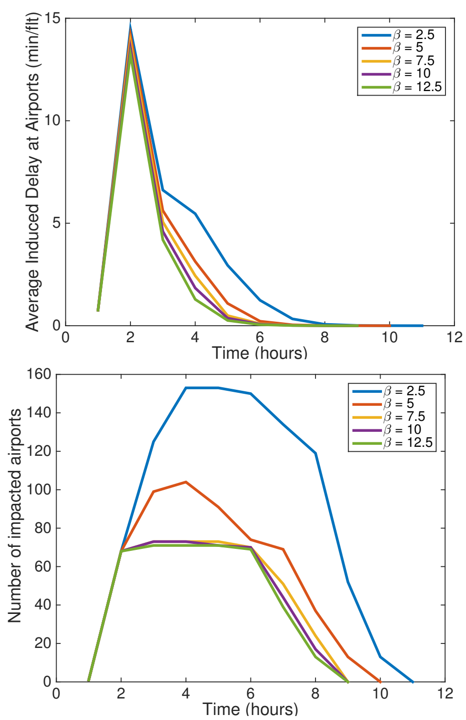

Fig. 4: (Top) Avg. induced delay, and (bottom) number of impacted airports for varying β, for an impulse input of 120 min/flight at ORD and $\alpha = 0.2$

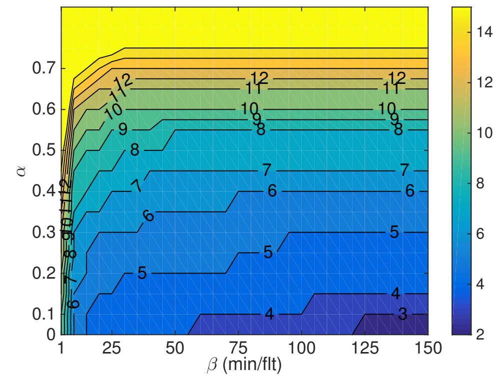

Fig. 5: Contour plot showing the time needed (in hours) for delays to subside, for varying values of α and β, for an impulse input delay of 120 min/flight at ORD

needed for delays to subside, for varying values of α and β, for an impulse input of 120 min/flight at ORD.

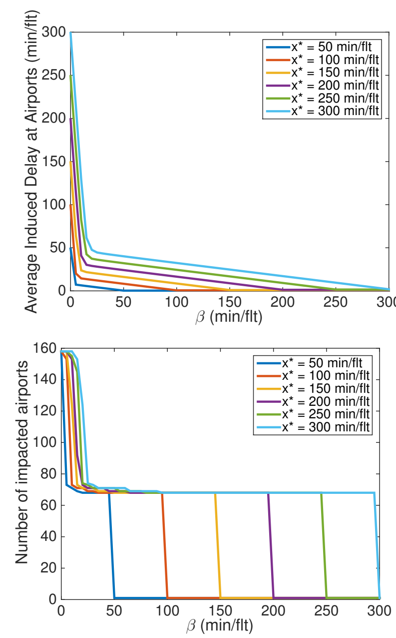

Fig. 6: System performance for different set-points x^*_{ORD}. Variation of (top) average induced delay and (bottom) number of impacted airports, with β

For a given set point x^*, the average induced delay as well as the number of airports impacted in the network decrease with increasing β (Fig 6). There are two distinct regions of decrease for the average induced delay plot. For low β values, the decrease is primarily due to the smaller number of airports impacted. At higher β values, the number of impacted airports does not change, and the decrease happens only at all those airports that are directly connected to the delay-inducing airport. When $\beta \geq x^*$, no delay gets transmitted. It is interesting to note that for a wide range of x^*, there is little benefit (in terms of delay) of investing in a β greater than 30 min/flight. The delays are sensitive to the slack in the system when the slack is small (that is, the schedules are very constrained).

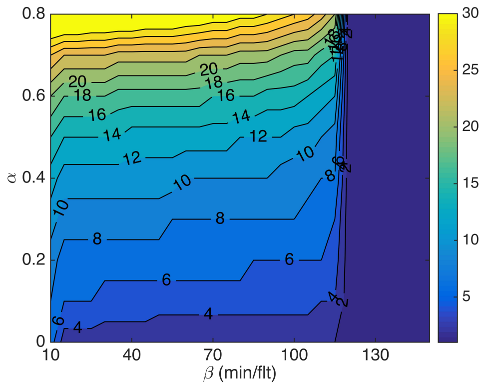

Fig. 7: Contour plot showing the time needed (in hours) for delays to subside, for varying values of α and β, for a set-point delay of 120 min/flight at ORD

Variations in β not only change the average induced delay, but also the geographical spread of the delay (Fig 8). When $\beta = 5$ min/flight, the exogenous input at ORD induces delays at many airports ranging from Seattle in the west coast to Miami in the south. With $\beta = 10$ min/flight, the delay gets limited to those airports that have a high fraction of their traffic coming directly from ORD.

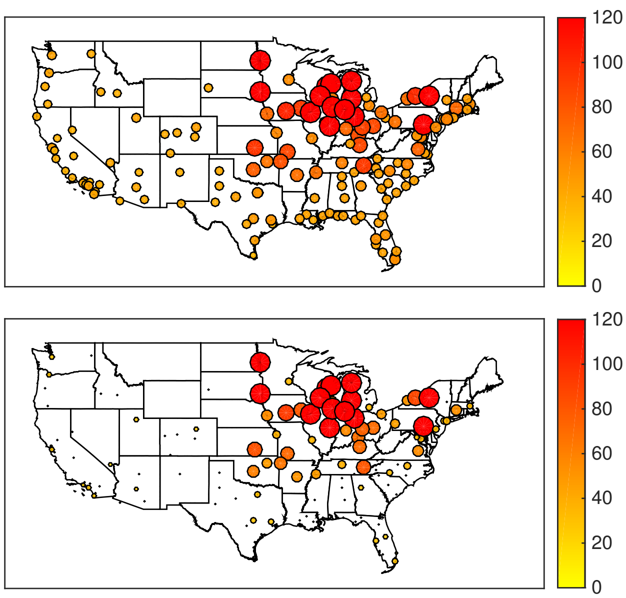

Fig. 8: System performance with a set-point $x^*_{ORD} = 120$ min/flight, (top) $\beta = 5$ min/flight and (bottom) $\beta = 10$ min/flight. We note the difference in both the number airports impacted, and the induced delays

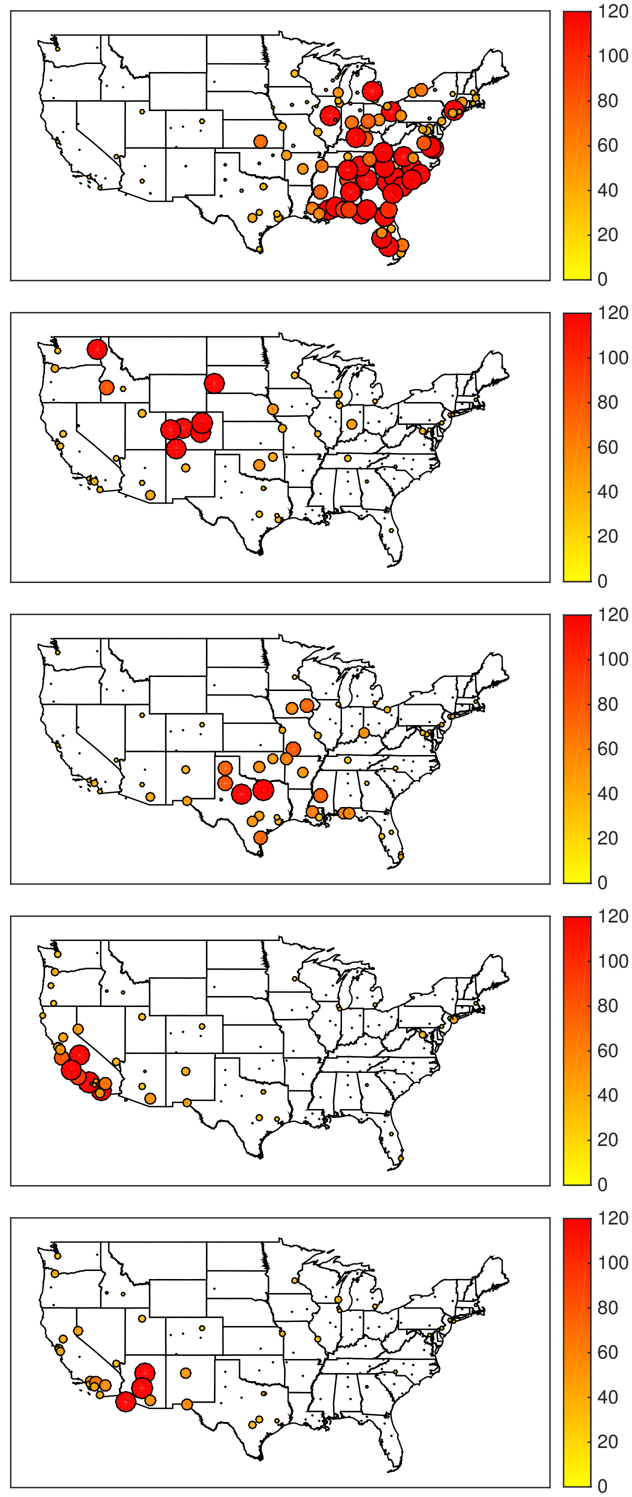

Fig. 9: Impact on the system when there is a set-point of 120 min/flight at (from top to bottom) ATL, DEN, DFW, LAX, and LAS

This analysis of the induced delay is extended to all airports. For each of the 158 airports in the data set, we use the appropriate exogenous input so that the set-point (delay level) is maintained at 120 min/flight. The 10 airports which can induce the highest delays throughout the system are shown in Table II.

TABLE II
TOP 10 AIRPORTS BY AVERAGE INDUCED DELAY WHEN $\beta = 10$ AND SET-POINT $x^* = 120$ MIN/FLIGHT AT THE INDUCING AIRPORT

Inducing Airport	Average Induced Delay(min/flight)
Atlanta (ATL)	31.75
Chicago (ORD)	17.82
Denver (DEN)	8.30
Dallas (DFW)	7.97
Los Angeles (LAX)	7.28
Phoenix (PHX)	5.42
San Francisco (SFO)	4.73
Baltimore (BWI)	4.37
Houston (IAH)	4.25
Honolulu (HNL)	3.45

TABLE III
TOP 10 AIRPORTS BY TOTAL NUMBER OF IMPACTED AIRPORTS WHEN $\beta = 10$ AND SET-POINT $x^* = 120$ MIN/FLIGHT AT THE INDUCING AIRPORT

Inducing Airport	Number of impacted airports
Atlanta (ATL)	125
Chicago (ORD)	74
Los Angeles (LAX)	68
Dallas (DFW)	40
Denver (DEN)	39
San Francisco (SFO)	52
Phoenix (PHX)	45
Houston (IAH)	40
Boston (BOS)	37
Orlando (MCO)	32

It is worth noting that the none of the airports in the New York area (EWR, JFK or LGA) appear to be significant in Tab. II or Tab. III. The reasons for these are several: First, both Newark (EWR) and John F. Kennedy (JFK) airports serve large numbers of international flights, which are not included in this analysis. Second, the three airports serve different airline networks, and their connectivity is quite diffused. However, it is worth noting that if the three airports were treated as a single "super-airport", then it would rank 6th in terms of induced delay (just below LAX in Table II) and tie for 8th place in terms of the number of airports impacted (i.e., tied with Houston).

Finally, we note that network effects can cause the induced delays to increase super-linearly with the degree of the node in the network (Fig. 10). This observation provides further rationale on why merging multiple airports (such as in the New York area) would serve to increase the degree of the network, and thereby significantly increase the induced delays. As expected, we also see that the induced delays decrease as the slack β increases.

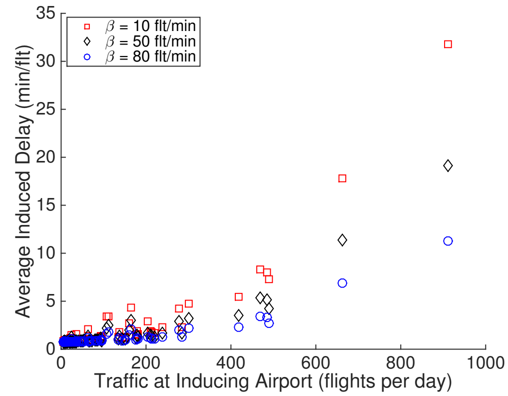

Fig. 10: Induced delay increases super-linearly with the degree of the node i, where $x_i^* = 120$ min/flight

IV. SUMMARY AND FUTURE WORK

This paper motivated and proposed a new model of delay propagation in air traffic networks. The model accounted for the tendency of delays to propagate through traffic flows at airports, while also accounting for the persistence of delays at an airport, the propagation of delays through the network, and the potential mitigation of delays due to slacks in the schedules. Disruptions were introduced in the form of an exogenous input, and a proof-of-concept illustration with BTS data was provided. The paper has shown the potential for such models to reflect system behavior, as measured by metrics such as, the amount of system-wide delay induced by disruptions at a particular airport, and the number of airports that are impacted when a given airport experiences disruptions.

In ongoing work, we are investigating the generalization of this approach to other data sets, as well as its ability to account for differences in schedule slack, inertia, etc. between different airports. In addition, we are investigating ways in which these models can be estimated and validated using operational data [15]. Finally, we are considering the case of linear networked systems under time-varying network topologies, and developing analysis tools for such systems.

REFERENCES

[1] Bureau of Transportation Statistics, "Airline On-Time Statistics and Delay Causes," 2015. [Online]. Available: http://www.transtats.bts.gov/

[2] M. Salathe, M. Kazandjieva, J. W. Lee, P. Levis, M. W. Feldman, and J. H. Jones, "A high resolution human contact network for infectious disease transmission," *Proceedings of the National Academy of Sciences*, vol. 107, no. 51, pp. 22 020–22 025, 2010.

[3] K. Dietz, "Epidemics and rumours: A survey," *Journal of the Royal Statistical Society. Series A (General)*, vol. 130, no. 4, pp. 505–528, 1967.

[4] R. Albert, I. Albert, and G. Nakarado, "Structural vulnerability of the North American power grid," *Physical Review E*, vol. 69, p. 025103, 2004.

[5] P. Crucitti, V. Latora, and M. Marchiori, "A topological analysis of the Italian electric power grid," *Physica A: Statistical Mechanics and its Applications*, vol. 338, pp. 92–97, 2004.

[6] M. Crovella and B. Krishnamurthy, *Internet Measurement: Infrastructure, Traffic and Applications*, 2006.

[7] V. Kalapala, V. Sanwalani, A. Clauset, and C. Moore, "Scale invariance in road networks," *Physical Review E*, vol. 73, p. 026130, 2006.

[8] C. von Ferber, Y. H. T. Holovatch, and V. Palchykov, "Public transport networks: empirical analysis and modeling," *The European Physical Journal B*, vol. 68, pp. 261–275, 2009.

[9] J. Manitz, "Statistical Inference for Propagation Processes on Complex Networks," Ph.D. dissertation, Georg-August-Universität Göttingen, 2014.

[10] M. Zanin and F. Lillo, "Modelling the air transport with complex networks: A short review," *The European Physical Journal Special Topics*, vol. 215, pp. 5–21, 2013.

[11] G. Spiers, P. Wei, and D. Sun, "Algebraic connectivity maximization of the air transportation network," in *American Control Conference*, 2012.

[12] P. Wei, G. Spiers, and D. Sun, "Algebraic connectivity maximization for air transportation networks," *IEEE Transactions on Intelligent Transportation Systems*, no. 2, pp. 685–698, 2014.

[13] O. Lordan, J. M. Sallan, P. Simo, and D. Gonzalez-Prieto, "Robustness of the air transport network," *Transportation Research Part E*, pp. 155–163, 2014.

[14] C. Nowzari, V. Preciado, and G. Pappas, "Analysis and Control of Epidemics: A Survey of Spreading Processes on Complex Networks," *IEEE Control Systems Magazine*, February 2016.

[15] K. Gopalakrishnan, H. Balakrishnan, and R. Jordan, "Clusters and Communities in Air Traffic Delay Networks," in *American Control Conference*, July 2016.

[16] N. Xu, K. B. Laskey, G. Donohue, and C. H. Chen, "Estimation of Delay Propagation in the National Aviation System Using Bayesian Networks," in *6th USA/Europe Air Traffic Management Research and Development Seminar*, June 2005.

[17] S. AhmadBeygi, A. Cohn, Y. Guan, and P. Belobaba, "Analysis of the Potential for Delay Propagation in Passenger Airline Networks," *Journal of Air Transport Management*, vol. 14 No. 5, pp. 221–236, 2008.

[18] M. Jetzki, "The propagation of air transport delays in Europe," Master's thesis, Department of Airport and Air Transportation Research, Aachen University, 2009.

[19] P. Fleurquin, J. Ramasco, and V. Eguiluz, "Systemic delay propagation in the US airport network," *Scientific Reports*, p. 1159, 2013.

[20] L. Hao, M. Hansen, Y. Zhang, and J. Post, "New York, New York: Two ways of estimating the delay impact of New York airports," *Transportation Research Part E: Logistics and Transportation Review*, vol. 70, pp. 245 – 260, 2014.

[21] A. Cook, G. Tanner, S. Cristóbal, and M. Zanin, "Delay propagation new metrics, new insights," in *11th USA/Europe Air Traffic Management Research and Development Seminar (ATM2011)*, Lisbon, Portugal, June 2015.

[22] P. Fleurquin, J. Ramasco, and V. Eguiluz, "Data-driven modeling of systemic delay propagation under severe meteorological conditions," in *11th USA/Europe Air Traffic Management Research and Development Seminar (ATM2011)*, Lisbon, Portugal, June 2015.

[23] R. Beatty, R. Hsu, L. Berry, and J. Rome, "Preliminary evaluation of flight delay propagation through an airline schedule," *Air Traffic Control Quarterly*, vol. 7, no. 4, pp. 259–270, 1999.

[24] A. Churchill, D. Lovell, and M. Ball, "Examining the temporal evolution of propagated delays at individual airports: case studies," in *Proceedings of the 7th USA/Europe ATM 2007 R&D Seminar, Barcelona, Spain, July*, 2007.

[25] ——, "Flight delay propagation impact on strategic air traffic flow management," *Transportation Research Record: Journal of the Transportation Research Board*, no. 2177, pp. 105–113, 2010.

[26] C. Ciruelos, A. Arranz, I. Etxebarria, S. Peces, B. Campanelli, P. Fleurquin, V. Eguiluz, and J. Ramasco, "Modelling delay propagation trees for scheduled flights."

[27] A. Klein, S. Kavoussi, D. Hickman, D. Simenauer, M. Phaneuf, and T. MacPhail, "Predicting Weather Impact on Air Traffic," in *Integrated Communication, Navigation and Surveillance (ICNS) Conference*, May 2007.

[28] B. Sridhar and N. Chen, "Short term national airspace system delay prediction," *Journal of Guidance, Control, and Dynamics*, vol. 32, No. 2, 2009.

[29] A. Klein, C. Craun, and R. S. Lee, "Airport delay prediction using weather-impacted traffic index (WITI) model," in *Digital Avionics Systems Conference (DASC)*, 2010.

[30] R. Yao, W. Jiandong, and X. Tao, "A flight delay prediction model with consideration of cross-flight plan awaiting resources," in *International Conference on Advanced Computer Control (ICACC)*, 2010.

[31] J. J. Rebollo and H. Balakrishnan, "Characterization and prediction of air traffic delays," *Transportation Research Part C*, pp. 231–241, 2014.

[32] Federal Aviation Administration (FAA), "Aviation System Performance Metrics (ASPM) website," Accessed 2016, http://aspm.faa.gov/.

[33] ——, "OPSNET website," Accessed 2016, http://aspm.faa.gov/.